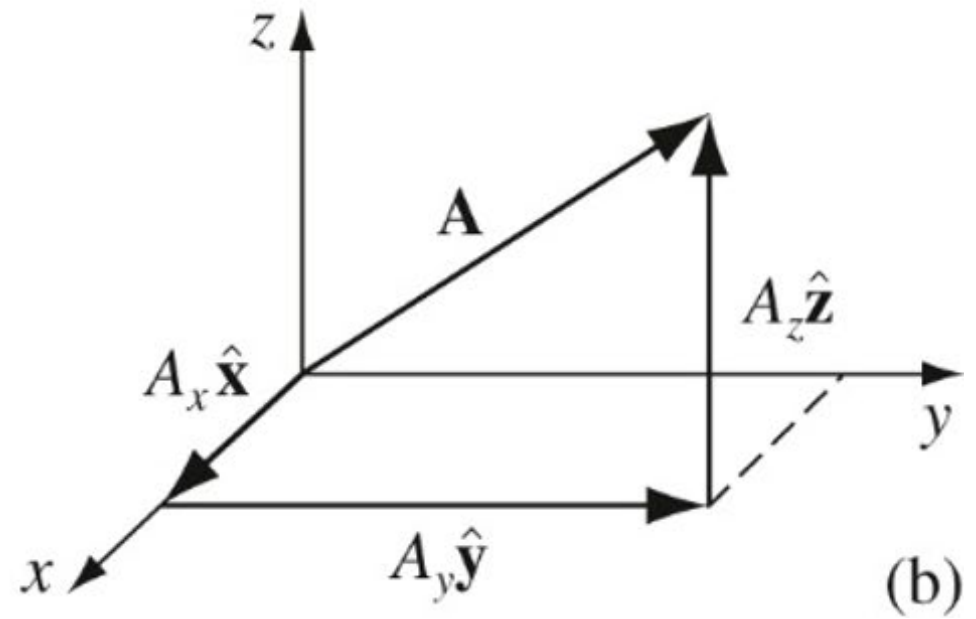
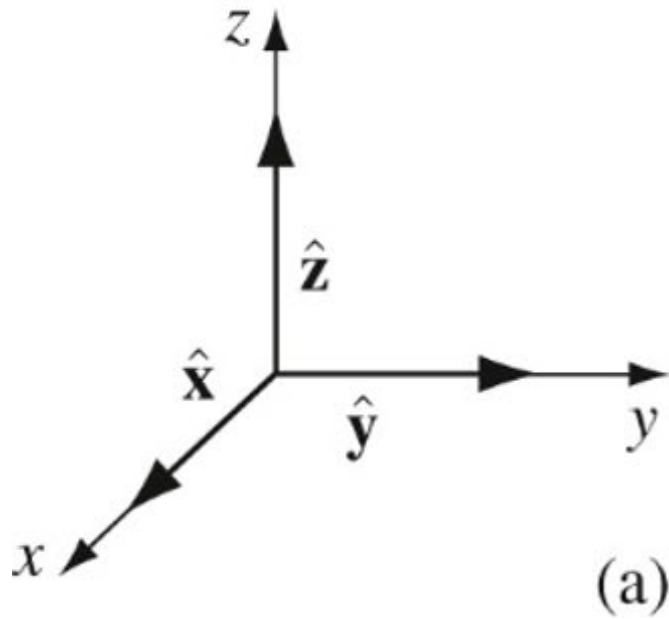


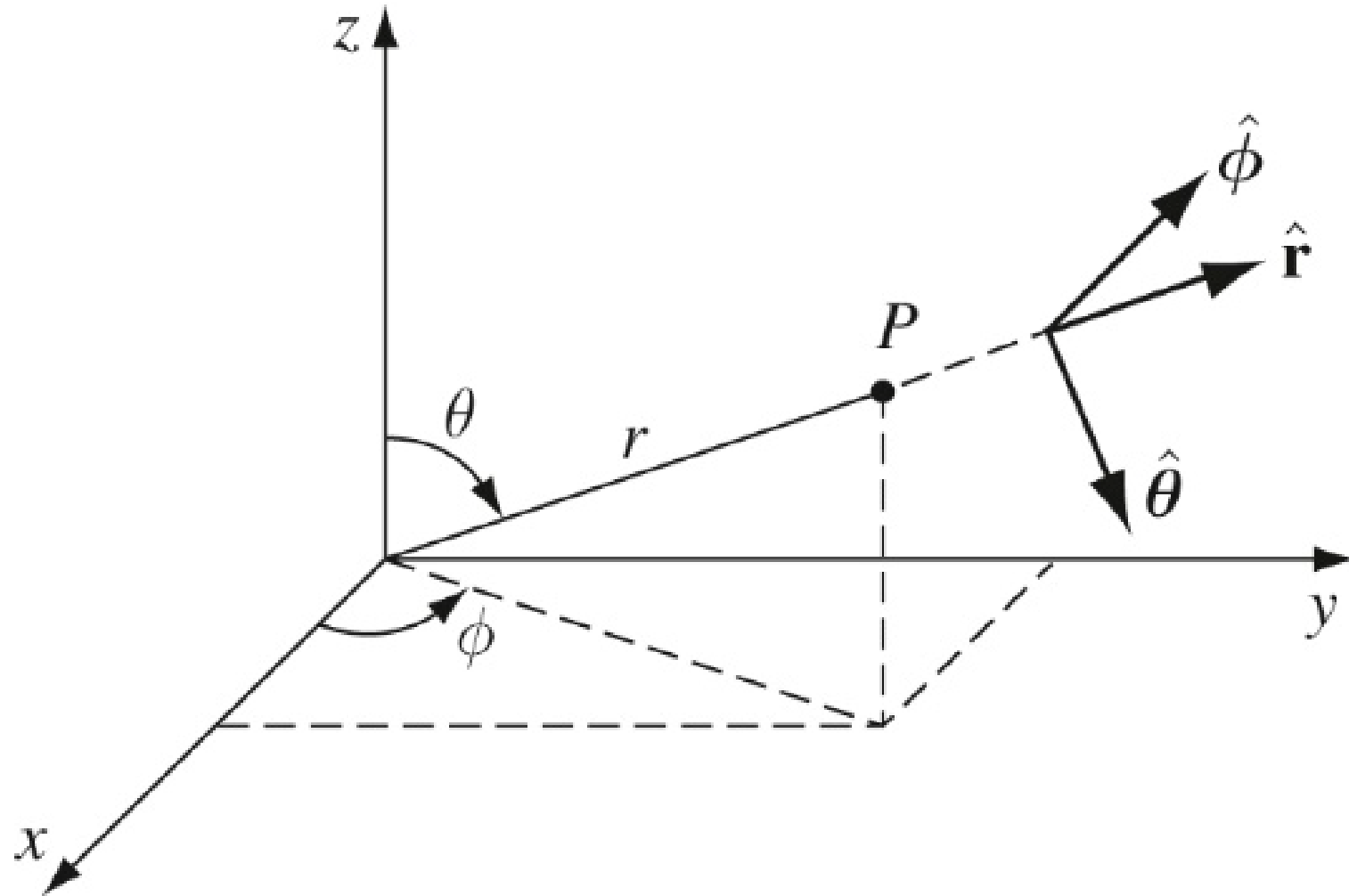
Lecture 2 Slides

Phys 313 Fall 2026

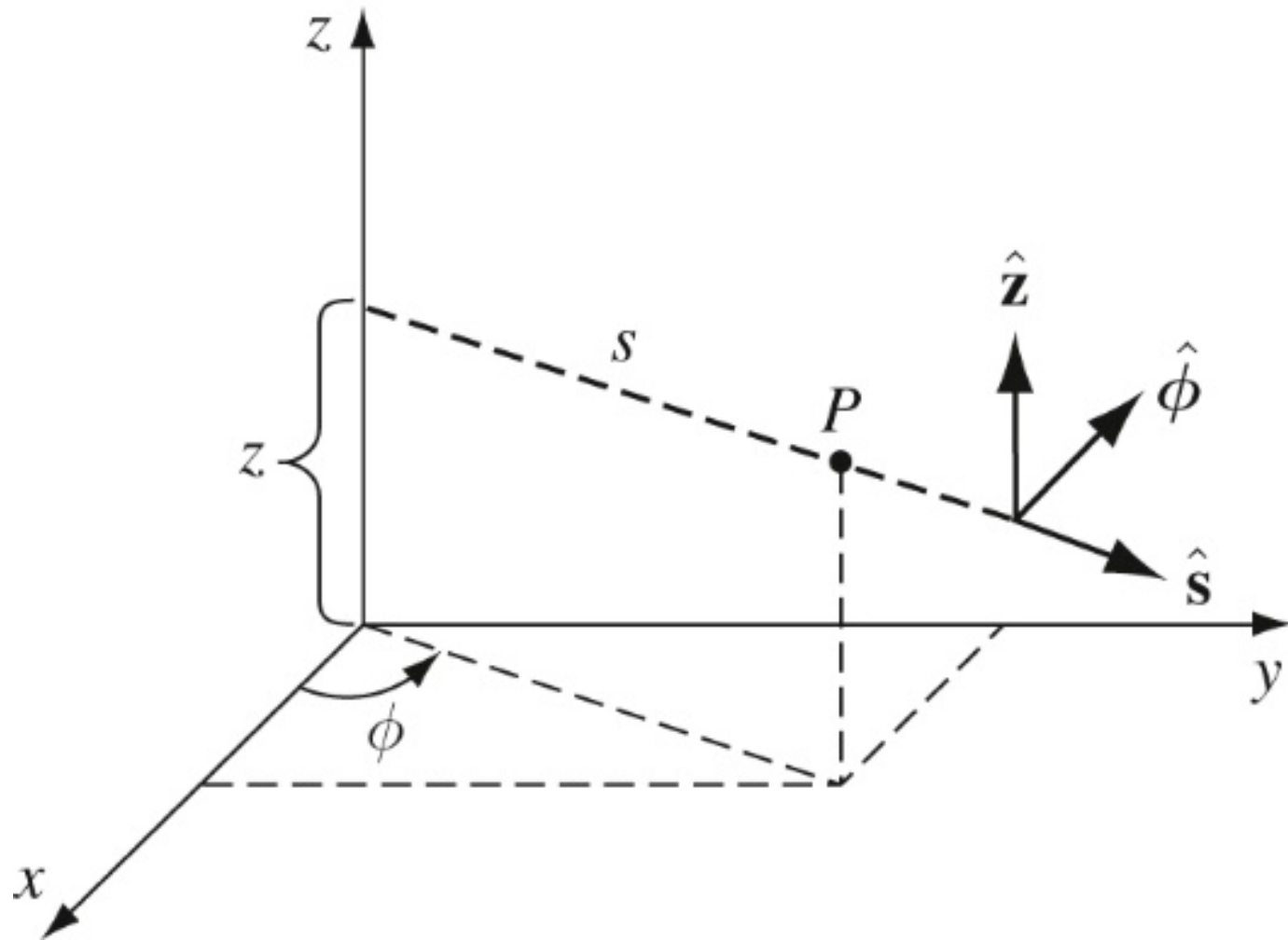
Cartesian Coordinate System



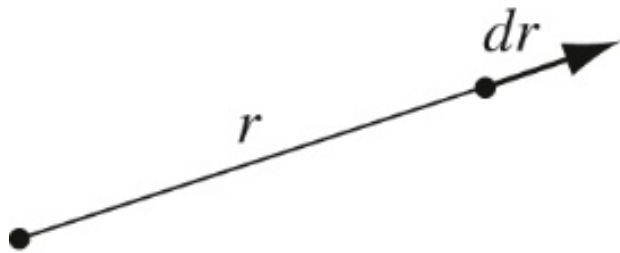
Spherical Coordinate System



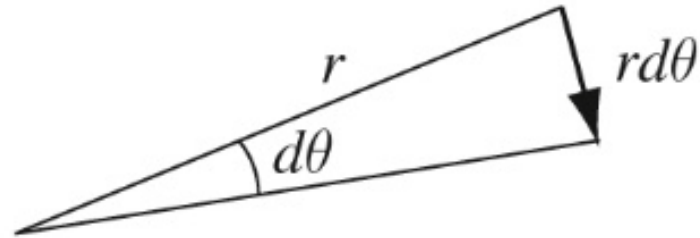
Cylindrical Coordinate System



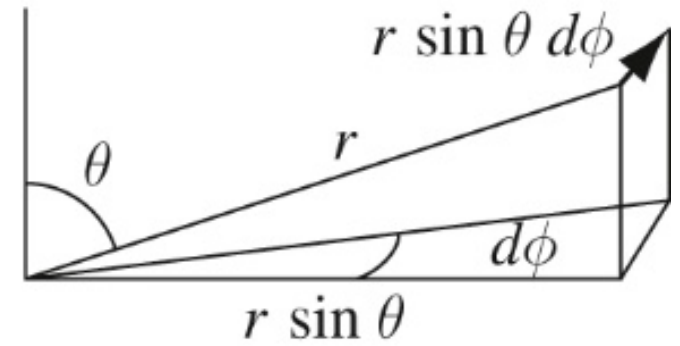
Differential Volume Element in Spherical Coordinates



(a)



(b)



(c)

Grad, Divergence, Curl and Laplacian in Spherical Coordinates

For a scalar field $f(r, \theta, \phi)$:

$$\nabla f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi}$$

For a vector field $\mathbf{A} = A_r \hat{r} + A_\theta \hat{\theta} + A_\phi \hat{\phi}$:

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\begin{aligned} \nabla \times \mathbf{A} = & \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right] \hat{r} \\ & + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right] \hat{\theta} \\ & + \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \hat{\phi} \end{aligned}$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

Grad, Divergence, Curl and Laplacian in Cylindrical Coordinates

For a scalar field $f(s, \phi, z)$:

$$\nabla f = \frac{\partial f}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}$$

For a vector field $\mathbf{A} = A_s \hat{s} + A_\phi \hat{\phi} + A_z \hat{z}$:

$$\nabla \cdot \mathbf{A} = \frac{1}{s} \frac{\partial}{\partial s} (s A_s) + \frac{1}{s} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\begin{aligned} \nabla \times \mathbf{A} = & \left[\frac{1}{s} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] \hat{s} \\ & + \left[\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} \right] \hat{\phi} \\ & + \frac{1}{s} \left[\frac{\partial}{\partial s} (s A_\phi) - \frac{\partial A_s}{\partial \phi} \right] \hat{z} \end{aligned}$$

$$\nabla^2 f = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial f}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$